Caringbah High School

Year 12 2021
Mathematics Extension 2
HSC Course
Assessment Task 4

General Instructions

- Reading time - 10 minutes
- Working time - 180 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11-16, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt Questions 11-16
- Allow about 2 hours and 45 minutes for this section

Marker's Use Only							
Section I	**Section II**						**Total**
Q 1-10	**Q11**	**Q12**	**Q13**	**Q14**	**Q15**	**Q16**	
/10	/15	/15	/15	/13	/15	/15	/98

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

1. In the Argand diagram the locus of points representing the complex number z such that $|z - 1 + i| = 9$ is a circle. What are the centre and radius of this circle?

(A) Centre (1, –1) and radius 3

(B) Centre (–1, 1) and radius 3

(C) Centre (–1, 1) and radius 9

(D) Centre (1, –1) and radius 9

2. The points A and B have position vectors of $\underset{\sim}{a} = \overrightarrow{OA}$ and $\underset{\sim}{b} = \overrightarrow{OB}$.
Point C is the midpoint of AB. Which expression is $\overrightarrow{OC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$?

(A) $\frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$

(B) $\frac{1}{2}(\underset{\sim}{a} - \underset{\sim}{b})$

(C) $\frac{1}{2}\underset{\sim}{a} + \underset{\sim}{b}$

(D) $\frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a})$

3. Which of the following is an expression for $\int \frac{x}{\sqrt{25 - x^2}} dx$?

(A) $-2\sqrt{25 - x^2} + C$

(B) $-\frac{1}{2}\sqrt{25 - x^2} + C$

(C) $-\sqrt{25 - x^2} + C$

(D) $\frac{1}{2}\sqrt{25 - x^2} + C$

4. What is the modulus and argument of $-1 - i$?

(A) Modulus $\sqrt{2}$ and argument $-\frac{\pi}{4}$

(B) Modulus $\sqrt{2}$ and argument $-\frac{3\pi}{4}$

(C) Modulus 2 and argument $-\frac{\pi}{4}$

(D) Modulus 2 and argument $-\frac{3\pi}{4}$

5. Which of the following expressions is equal to $\int 2e^x \cos x dx$?

(A) $-2e^x(\sin x + \cos x) + C$

(B) $-e^x(\sin x + \cos x) + C$

(C) $2e^x(\sin x + \cos x) + C$

(D) $e^x(\sin x + \cos x) + C$

6. A particle moving in a straight line obeys $v^2 = 4(5 + 4x - x^2)$ where x is its displacement from the origin in metres and v is its velocity in ms^{-1}. Where is the centre of the motion?

(A) $x = -1$

(B) $x = 0$

(C) $x = 2$

(D) $x = 5$

7. The velocity of a particle moving in a straight line is given by $v = 2 - x$ where x metres is the distance from fixed point O, v is the velocity in metres per second and a is the acceleration in metres per second per second. Initially the particle is at O.
Which expression is a in terms of x ?

(A) $a = x - 2$

(B) $a = x - 4$

(C) $a = \frac{1}{2}(x - 2)$

(D) $a = \frac{1}{2}(x - 4)$

8. A vector of magnitude $\sqrt{736}$ and with direction opposite to $2\underset{\sim}{i} - 4\underset{\sim}{j} + 4\underset{\sim}{k}$ is:

(A) $12\underset{\sim}{i} - 24\underset{\sim}{j} + 24\underset{\sim}{k}$

(B) $4\underset{\sim}{i} - 8\underset{\sim}{j} + 8\underset{\sim}{k}$

(C) $-12\underset{\sim}{i} + 24\underset{\sim}{j} - 4\underset{\sim}{k}$

(D) $-6\underset{\sim}{i} + 12\underset{\sim}{j} - 12\underset{\sim}{k}$

9. What is the negation of the statement: $n > 1$ or $n < -1$?

(A) $n < 1$ or $n > -1$

(B) $n < 1$ and $n > -1$

(C) $n \leq 1$ or $n \geq -1$

(D) $n \leq 1$ and $n \geq -1$

10. A recurrence relation is defined below:

$$I_n = \int_0^1 \frac{1}{(1+x^2)^n}\,dx \qquad \text{n} = 1,2,3,\ldots$$

$$I_{n+1} = \frac{2n-1}{2n} I_n + \frac{1}{n \times 2^{n+1}} \qquad n = 1,2,3,\ldots$$

The exact value of $\displaystyle\int_0^1 \frac{1}{(1+x^2)^4}\,dx$ is ?

(A) $\dfrac{15\pi + 33}{192}$

(B) $\dfrac{3\pi + 8}{32}$

(C) $\dfrac{5}{8}$

(D) $\dfrac{11}{32}$

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) Two vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are given by $\underset{\sim}{u}$ and $\underset{\sim}{v}$.

$\underset{\sim}{u} = \underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}, \qquad \underset{\sim}{v} = -\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$

(i) Find $\underset{\sim}{v} - \underset{\sim}{u}$ **1**

(ii) Find $4\underset{\sim}{u} \cdot \underset{\sim}{v}$ **2**

(iii) Interpret the result in part (ii) geometrically. **1**

(b) **(i)** Suppose that a and b are real non-negative numbers. **2**
Prove that $a + b > 2\sqrt{ab}$.

(ii) Hence prove that $(a+b)(a+c)(b+c) \geq 8abc$, where c is also a real non-negative number. **2**

(c) Let $z_1 = \cos\theta_1 + i\sin\theta_1$ and $z_2 = \cos\theta_2 + i\sin\theta_2$ where θ_1 and θ_2 are real.
Show that:

(i) $\dfrac{1}{z_1} = \cos\theta_1 - i\sin\theta_1$ **1**

(ii) $z_1 z_2 = \cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)$ **1**

(d) The point P has position vector $\overrightarrow{OP} = -2\underset{\sim}{i} + 3\underset{\sim}{j} - 5\underset{\sim}{k}$ relative to an origin O. **2**
Find a unit vector parallel to $\overrightarrow{OP}$.

(e) Find the square root of $z = -5 + 12i$. **3**

End of Question 11

Question 12 (15 marks) **Marks**

(a) By completing the square, find $\displaystyle\int \frac{1}{x^2 - 6x + 10}\,dx$ **2**

(b) Find all real x such that $|x - 2| > \sqrt{x}$. **3**

(c) Show that **2**

$$\int_0^{\frac{\pi}{2}} e^{(1+\sin x)} \cos x \, dx = e(e - 1)$$

(d) The point *A* has position vector $2\underset{\sim}{i} + 6\underset{\sim}{j} - \underset{\sim}{k}$ and point *B* has position vector $3\underset{\sim}{i} + 4\underset{\sim}{j} + \underset{\sim}{k}$, relative to the origin. The line ℓ_1 passes through the points *A* and *B*.

(i) Find the vector $\overrightarrow{AB}$. **1**

(ii) Find a vector equation for the line ℓ_1. **1**

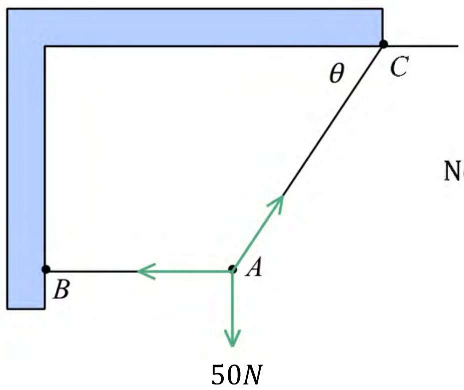
(iii) A second line ℓ_2 passes through the origin and is parallel to the vector $\underset{\sim}{i} + \underset{\sim}{k}$. Find the acute angle, (**in radians**), between ℓ_1 and ℓ_2. **3**

(iv) Line ℓ_1 meets line ℓ_2 at the point *C*. What is position vector of point *C* ? **3**

End of Question 12

Question 13 (15 marks) **Marks**

(a) A particle *A* with a tension of 50N is held in equilibrium by two strings, *AB* and *AC*. *AB* is attached to the wall and is horizontal. *AC* is attached to the ceiling and makes an angle of θ with the horizontal, as shown in the diagram below.
The tension in *AC* is twice the tension in *AB*.



(i) Find the value of θ. **2**

(ii) Calculate the tensions in the strings AB and AC. **2**
Answer correct to one decimal place.

(b) **(i)** Find real numbers *A*, *B* and *C* such that **2**

$$\frac{x^2}{4x^2-9} = \text{A} + \frac{B}{2x-3} + \frac{C}{2x+3}$$

(ii) Hence find $\int \frac{x^2}{4x^2-9}dx$ **2**

(c) A particle is moving in Simple Harmonic Motion (SHM) with acceleration $\frac{d^2y}{dx^2} = -4x \text{ ms}^{-2}$.
If the particle starts at the origin with a velocity of 3 ms^{-1}, find:

(i) the endpoints of its motion **3**

(ii) the exact speed when the particle is 1 m from the origin **1**

(d) The point *P* has position vector $\overrightarrow{OP} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$, point *Q* has position vector $\overrightarrow{OQ} = -4\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k}$ and point *R* has position vector $\overrightarrow{OR} = 3\underset{\sim}{i} + 4\underset{\sim}{j} - 3\underset{\sim}{k}$ relative to an origin *O*. The point *S* is such that *PQRS* is a parallelogram. Find the position vector of point *S*. **3**

End of Question 13

Question 14 (13 marks) **Marks**

(a) Use the substitution $t = \tan x$ to evaluate the following correct to 3 significant figures: **4**

$$\int_0^{\frac{\pi}{4}} \frac{1}{3\sin^2 x + 5\cos^2 x}\,dx$$

(b) A particle's motion is simple harmonic in a straight line. At time *t* seconds its displacement from a fixed point *O* in the line is *x* metres, given by:

$$x = 1 + \sqrt{2}\cos\left(t - \frac{\pi}{4}\right)$$

(i) Show that $\ddot{x} = -(x-1)$ **1**

(ii) Find the time taken for the particle to first pass through the point *O*. **2**

(iii) Find in simplest exact form, the average speed of the particle during one complete oscillation of its motion. **2**

(c) Given $z = \sqrt{2} + 4i$ and $w = 2 + bi,$ where $b \in \mathbb{R},$ is such that $\arg w = \frac{1}{4}\pi.$

(i) Show that $b = 2$ **1**

(ii) Calculate $|zw|$ **1**

(iii) Find the value of *n* for which $16|z^n| = 81|w^n|$ **2**

End of Question 14

Question 15 (15 marks) **Marks**

(a) On an Argand diagram mark the points P, Q representing the complex numbers $z_1 = 4 + i$ and $z_2 = 1 + 4i$ respectively.

(i) Show how to construct the point R representing $z_1 + z_2$. **2**

(ii) What type of quadrilateral is $OPQR$ where O is the origin? Explain. **1**

(b) **(i)** Let $I_n = \int x^n e^x dx$ **2**

Show that $I_n = x^n e^x - nI_{n-1}$ for $n = 1, 2, 3, \ldots$

(ii) Hence or otherwise, find the exact value of $\int_0^2 x^2 e^x dx$ **2**

(c) A projectile is fired from a point O with speed V ms^{-1} at an angle θ above the horizontal, where $0 < \theta < \frac{\pi}{2}$. The projectile moves in a vertical plane under gravity where the acceleration due to gravity is 10 ms^{-2}.

(i) Show that after t seconds, the horizontal and vertical displacements of the projectile are given by $x = Vt\cos\theta$ and $y = -5t^2 + Vt\sin\theta$. **2**

(ii) Derive an expression for the Cartesian equation of motion. **2**

(iii) If the projectile has an initial velocity of 60 ms^{-1} it just clears a 2 metre high wall 25 metres from the point of projection. The base of the wall is at the same level as the point of projection. Calculate the angle(s) of projection to the nearest minute. **2**

(d) If $(x - 2)^2$ is an odd integer, prove by contrapositive that x is even. **2**

End of Question 15

Question 16 (15 marks) **Marks**

(a) Suppose that $z = \frac{1}{2}e^{i\theta}$ where θ is real.

(i) Find $|z|$ **1**

(ii) Show that the imaginary part of the geometric series **3**

$$1 + z + z^2 + \cdots = \frac{1}{1 - z}$$

can be expressed as $\dfrac{2\sin\theta}{5 - 4\cos\theta}$.

(b) An object of mass 50 kg falls from a height under a gravitational acceleration of g meters per second squared and air resistance of $0.05v$ newtons.

(i) Show that $t = 1000 \ln\left(\dfrac{g}{g - 0.001v}\right)$. **3**

(ii) Find the velocity of the object after time *t*. **1**

(iii) What is the limiting (terminal) velocity of the object? **1**

(iv) How far has the object fallen after time *t*. **2**

(c) Consider the equation $x^3 - 3x - 1 = 0$. **4**

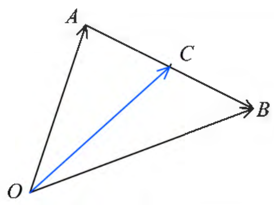
Let $x = \frac{p}{q}$ where p and q are integers having no common divisors other than ± 1.

Suppose that x is a root of the equation $ax^3 - 3x + b = 0$, where a and b are integers.

Explain why p divides b and why q divides a and deduce that $x^3 - 3x - 1 = 0$ does not have a rational root.

End of paper

Year 12 Mathematics Extension 2 Yearly Examination
Worked Solutions and Marking Guidelines

Section I		
	Solution	**Criteria**
1	$\lvert z-1+i\rvert=9$ $\lvert z-(1-i)\rvert=9$ $\therefore$ Centre (1, –1) and radius 3.	1 Mark: D
2	$\overrightarrow{CB}=\frac{1}{2}\overrightarrow{AB}$ $=\frac{1}{2}(\underset{\sim}{b}-\underset{\sim}{a})$  	1 Mark: D
3	Let $u=16-x^2$ then $\frac{du}{dx}=-2x$ or $xdx=-\frac{1}{2}du$ $\int\frac{x}{\sqrt{25-x^2}}dx=-\frac{1}{2}\int u^{-\frac{1}{2}}du$ $=-\frac{1}{2}\times 2u^{\frac{1}{2}}+C$ $=-\sqrt{25-x^2}+C$	1 Mark: C
4	$\tan\theta=\frac{-1}{-1}$ $\theta=-\frac{3\pi}{4}$ $r^2=x^2+y^2$ $=(-1)^2+(1)^2$ $r=\sqrt{2}$ $\therefore$ Modulus $\sqrt{2}$ and argument $-\frac{3\pi}{4}$	1 Mark: B
5	$\int 2e^x\cos x dx=\int 2e^x\times\frac{d}{dx}(\sin x)dx$ $=2e^x(\sin x)-\int 2e^x(\sin x)dx$ $=2e^x\sin x-\int 2e^x\times\frac{d}{dx}(-\cos x)dx$ $=2e^x\sin x-\left[2e^x(-\cos x)-\int 2e^x(-\cos x)dx\right]$ $2\int 2e^x\cos x dx=2e^x\sin x+2e^x\cos x+C$ $\therefore\int 2e^x\cos x dx=e^x(\sin x+\cos x)+C$	1 Mark: D

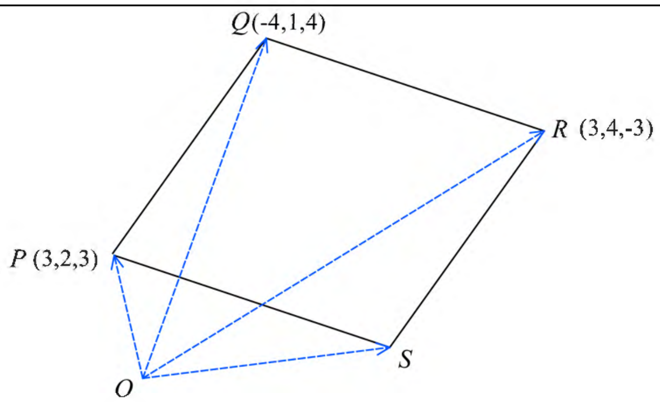
6	Simple harmonic motion occurs when $\ddot{x} = -n^2(x-b)$ $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ $= \frac{d}{dx}\left(\frac{1}{2} \times 4(5+4x-x^2)\right)$ $= \frac{4}{2} \times (-2x+4)$ $= -4(x-2)$ $\therefore$ Centre of motion $x = 2$.	1 Mark: C
7	$v = 2 - x$ $v^2 = 4 - 4x + x^2$ $\frac{1}{2}v^2 = 2 - 2x + \frac{1}{2}x^2$ $a = \frac{d}{dx}\left(2 - 2x + \frac{1}{2}x^2\right)$ $a = x - 2$	1 Mark: A
8	Vectors (C) and (D) are opposite direction to $2\underset{\sim}{i} - 4\underset{\sim}{j} + 4\underset{\sim}{k}$ (C) $\sqrt{(-12)^2 + 24^2 + (-4)^2} = \sqrt{736}$ (D) $\sqrt{(-6)^2 + 12^2 + (-12)^2} = \sqrt{324} = 18$	1 Mark: C
9	If P is a statement then the statement 'not P' is the negation of P. It is true precisely when the original statement is false. $\therefore n \leq 1$ and $n \geq -1$	1 Mark: D
10	$I_4 = \frac{5}{6}I_3 + \frac{1}{48}$ $I_3 = \frac{3}{4}I_2 + \frac{1}{16} = \frac{3}{4}\left(\frac{1}{2}I_1 + \frac{1}{4}\right) + \frac{1}{16} = \frac{3}{8}I_1 + \frac{1}{4}$ $\therefore I_4 = \frac{5}{6}\left(\frac{3}{8}I_1 + \frac{1}{4}\right) + \frac{1}{48} = \frac{5}{16}I_1 + \frac{11}{48}$ $I_1 = \int_0^1 \frac{1}{1+x^2}\,dx$ $= [\tan^{-1}x]_0^1 = \frac{\pi}{4}$ $\therefore \int_0^1 \frac{1}{(1+x^2)^4}\,dx = \frac{15\pi + 33}{192}$	1 Mark: A

Section II		
	Solution	**Criteria**
11(a) (i)	$\underset{\sim}{v} - \underset{\sim}{u} = (-\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}) - (\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k})$ $= -2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$	1 Mark: Correct answer.
11(a) (ii)	$4\underset{\sim}{u} \cdot \underset{\sim}{v} = 4(\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}) \cdot (-\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k})$ $= (4\underset{\sim}{i} + 8\underset{\sim}{j} - 4\underset{\sim}{k}) \cdot (-\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k})$ $= -4 + 8 - 4$ $= 0$	2 Marks: Correct answer. 1 Mark: Finds the scalar multiplication or scalar product.
11(a) (iii)	$3\underset{\sim}{u}$ and $\underset{\sim}{v}$ are perpendicular to each other.	1 Mark: Correct answer.
11(b) (i)	Let $a = x^2, b = y^2$ where $x, y \geq 0$ $a + b - 2\sqrt{ab} = x^2 + y^2 - 2xy$ $= (x - y)^2$ ≥ 0 Therefore $a + b \geq 2\sqrt{ab}$	2 Marks: Correct answer. 1 Mark: Constructs inequality.
11(b) (ii)	From part (i), we have $a + b \geq 2\sqrt{ab}$ and we know $2\sqrt{ab} \geq 0$. Similarly $a + c \geq 2\sqrt{ac} \geq 0$ and $b + c \geq 2\sqrt{bc} \geq 0$. Therefore $(a + b)(a + c)(b + c) \geq (2\sqrt{ab})(2\sqrt{ac})(2\sqrt{bc})$ $(a + b)(a + c)(b + c) \geq 8\sqrt{ab \times ac \times bc}$ $(a + b)(a + c)(b + c) \geq 8\sqrt{a^2b^2c^2}$ $(a + b)(a + c)(b + c) \geq 8abc$	2 Marks: Correct answer. 1 Mark: Constructs inequality.
11(c) (i)	$\frac{1}{z_1} = (\cos\theta_1 + i\sin\theta_1)^{-1}$ $= \cos(-\theta_1) + i\sin(-\theta_1)$ De Moivre's Theorem $= \cos\theta_1 - i\sin\theta_1$	1 Mark: Correct answer.
11(c) (ii)	$z_1z_2 = (\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$ $= \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + i(\sin\theta_1\cos\theta_2 + \sin\theta_2\cos\theta_1)$ $= \cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)$	1 Mark: Correct answer.
11(d)	$\lvert\overrightarrow{OP}\rvert = \sqrt{(-2)^2 + 3^2 + (-5)^2}$ $= \sqrt{38}$ $\widehat{OP} = \frac{\overrightarrow{OP}}{\lvert\overrightarrow{OP}\rvert}$ $= \frac{1}{\sqrt{38}}(-2\underset{\sim}{i} + 3\underset{\sim}{j} - 5\underset{\sim}{k})$	2 Marks: Correct answer. 1 Mark: Finds the magnitude of $\overrightarrow{OP}$.

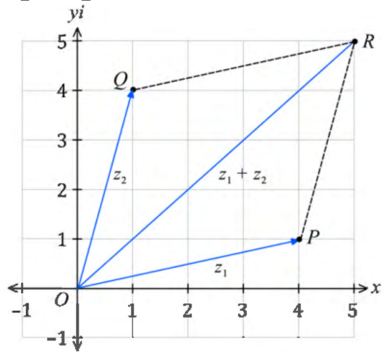
11(e)	Let $(a+ib)^2 = -5+12i$ where a and b are real numbers $(a^2-b^2)+(2ab)i = -5+12i$ Equating the real and imaginary parts $(a^2-b^2) = -5$ ① $2ab = +12$ $a = \frac{6}{b}$ ② Substituting equation ② into equation ① $\left(\frac{6}{b}\right)^2 - b^2 = -5$ $36 - b^4 = -5b^2$ $b^4 - 5b^2 - 36 = 0$ $(b^2-9)(b^2+4)$ Since b is real then $b = 3$ or $b = -3$ $\therefore a = -2, b = 3$ or $a = 2, b = -3$ Hence $z = -5+12i$ has square roots $-2-3i$ and $2+3i$.	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Uses $(a+ib)^2$ or shows some understanding.
12(a)	$\int \frac{1}{x^2-6x+10}dx = \int \frac{dx}{(x-3)^2+1^2}$ $= \tan^{-1}(x-3)+C$	2 Marks: Correct answer. 1 Mark: Completes the square.
12(b)	Inequality is only defined for $x \geq 0$ ① (cannot find the square root of a negative number) Using the result $\|x\| = \sqrt{x^2}$ or $\|x-2\| = \sqrt{(x-2)^2}$ $\sqrt{(x-2)^2} > \sqrt{x}$ $(x-2)^2 > x$ $x^2-4x+4 > x$ $x^2-5x+4 > 0$ $(x-1)(x-4) > 0$ $\therefore x < 1$ or $x > 4$ ② Combining results ① and ② $\therefore 0 \leq x < 1$ or $x > 4$	3 Marks: Correct answer. 2 Marks: Finds one correct region or makes significant progress. 1 Mark: Uses $\|x\| = \sqrt{x^2}$ or shows some understanding.
12(c)	$u = \sin x + 1$ $du = \cos x\, dx$ When $x = 0$ then $u = 1$ and when $x = \frac{\pi}{2}$ then $u = 2$ $\int_0^{\frac{\pi}{2}} e^{\sin x+1}\cos x\, dx = \int_1^2 e^u du$ $= [e^u]_1^2$ $= e^2 - e^1$ $= e(e-1)$	2 Marks: Correct answer. 1 Mark: Finds the primitive function or sets up the integration using substitution.

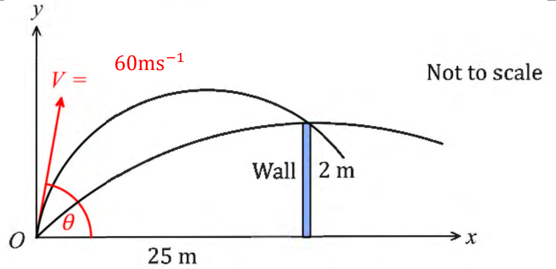
12(d) (i)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $= (3\underset{\sim}{i} + 4\underset{\sim}{j} + \underset{\sim}{k}) - (2\underset{\sim}{i} + 6\underset{\sim}{j} - \underset{\sim}{k})$ $= \underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$	1 Mark: Correct answer.
12(d) (ii)	Vector equations for l_1. $l_1 = (2\underset{\sim}{i} + 6\underset{\sim}{j} - \underset{\sim}{k}) + \lambda(\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k})$ or $l_1 = (3\underset{\sim}{i} + 4\underset{\sim}{j} + \underset{\sim}{k}) + \lambda(\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k})$	1 Mark: Correct answer.
12(d) (iii)	$l_1 = \underset{\sim}{u} = \underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$ $l_2 = \underset{\sim}{v} = (0\underset{\sim}{i} + 0\underset{\sim}{j} + 0\underset{\sim}{k}) + \mu(\underset{\sim}{i} + 0\underset{\sim}{j} + \underset{\sim}{k})$ $= \underset{\sim}{i} + 0\underset{\sim}{j} + \underset{\sim}{k}$ $\lvert\underset{\sim}{u}\rvert = \sqrt{1^2 + (-2)^2 + 2^2} = 3$ $\lvert\underset{\sim}{v}\rvert = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}$ $\underset{\sim}{u} \cdot \underset{\sim}{v} = (1 \times 1) + (-2 \times 0) + (2 \times 1) = 3$ $\cos\theta = \frac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{\lvert\underset{\sim}{u}\rvert\lvert\underset{\sim}{v}\rvert} = \frac{3}{3 \times \sqrt{2}} = \frac{1}{\sqrt{2}}$ $\theta = \frac{\pi}{4}$	3 Marks: Correct answer. 2 Marks: Uses the angle between two vectors. 1 Mark: Shows some understanding.
12(d) (iv)	Line l_1 intersects the line l_2 then: $(2\underset{\sim}{i} + 6\underset{\sim}{j} - \underset{\sim}{k}) + \lambda(\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}) = \mu(\underset{\sim}{i} + 0\underset{\sim}{j} + \underset{\sim}{k})$ $2 + \lambda = \mu$ ① $6 - 2\lambda = 0$ ② $-1 + 2\lambda = \mu$ ③ From equation ② $\lambda = 3$ From equation ① or ③ $\mu = 5$ Hence $(2\underset{\sim}{i} + 6\underset{\sim}{j} - \underset{\sim}{k}) + 3(\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}) = 5\underset{\sim}{i} + 5\underset{\sim}{k}$ or $5(\underset{\sim}{i} + 0\underset{\sim}{j} + \underset{\sim}{k}) = 5\underset{\sim}{i} + 5\underset{\sim}{k}$ $\therefore$ Position vector of point C is $5\underset{\sim}{i} + 5\underset{\sim}{k}$.	3 Marks: Correct answer. 2 Marks: Finds the correct values for λ and μ. 1 Mark: Shows some understanding.
13(a) (i)	Resolving horizontal forces at A $T_{AB} = T_{AC}\cos\theta$ $\cos\theta = \frac{T_{AB}}{T_{AC}}$ $= \frac{1}{2}$ $\theta = 60°$	2 Marks: Correct answer. 1 Mark: Resolves the forces horizontal.

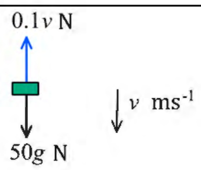
13(a) (ii)	Resolving vertical forces at *A* $T_{AC}\sin 60° = 50$ $T_{AC} = \dfrac{50}{\sin 60°}$ $= 57.7350\ldots$ $\approx 57.7\text{ N}$ $\dfrac{T_{AB}}{T_{AC}} = \dfrac{1}{2}$ $T_{AB} = T_{AC} \div 2$ $= 28.868\ldots$ $\approx 28.9\text{ N}$	2 Marks: Correct answer. 1 Mark: Resolves the forces vertically.
13(b) (i)	$\dfrac{x^2}{4x^2-9} = \text{A} + \dfrac{B}{2x-3} + \dfrac{C}{2\text{x}+3}$ Using partial fractions to find *A*, *B* and *C* $A(4x^2-9) + \text{B}(2x+3) + C(2x-3) = x^2$ $4Ax^2 - 9A + 2\text{B}x + 3B + 2Cx - 3C = x^2$ $4A = 1 \ \therefore \text{A} = \dfrac{1}{4}$ ① $2B + 2C = 0 \ \therefore B + C = 0$ ② $-9A + 3B - 3C = 0 \ \therefore B - \text{C} = \dfrac{3}{4}$ ③ Equation ② + ③ $2B = \frac{3}{4}$ $\therefore B = \dfrac{3}{8} \ \ \therefore C = -\dfrac{3}{8}$	2 Marks: Correct answer. 1 Mark: Makes progress in finding *A*, *B* or *C*.
13(b) (ii)	$\displaystyle\int \frac{x^2}{4x^2-9}\,dx = \int \frac{1}{4} + \frac{\frac{3}{8}}{2x-3} - \frac{\frac{3}{8}}{2\text{x}+3}\,dx$ $\displaystyle= \int \frac{1}{4}\,dx + \frac{3}{16}\int \frac{2}{2x-3}\,dx - \frac{3}{16}\int \frac{2}{2\text{x}+3}\,dx$ $= \dfrac{x}{4} + \dfrac{3}{16}\ln\lvert 2x-3\rvert - \dfrac{3}{16}\ln\lvert 2x+3\rvert + C$ $= \dfrac{x}{4} + \dfrac{3}{16}\ln\left(\left\lvert\dfrac{2x-3}{2x+3}\right\rvert\right) + C$	2 Marks: Correct answer. 1 Mark: Correctly finds one of the integrals.
13(c) (i)	$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4x$ $\displaystyle\left(\frac{1}{2}v^2\right) = \int -4x\,dx = \frac{-4x^2}{2} + C$ when $\text{x} = 0, v = 3 \ \therefore C = \dfrac{9}{2}$ $\therefore v = \sqrt{9-4x^2}$ $\therefore$ when $v = 0 \quad x = \pm\dfrac{3}{2}$ $\therefore$ endpoints are $x = \pm\dfrac{3}{2}$	3 marks: Correct answer. 2 mark: Incorrect answer of velocity. 1 mark: Finds integral of acceleration.

13(c)(ii)	$x = 1, v^2 = 9 - 4$ $\therefore v = \pm\sqrt{5}$ $\therefore speed = \sqrt{5}$	1 Mark: Correct answer.
13(d)	 $\overrightarrow{OP} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$, $\overrightarrow{OQ} = -4\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k}$, $\overrightarrow{OR} = 3\underset{\sim}{i} + 4\underset{\sim}{j} - 3\underset{\sim}{k}$ $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ $= (-4\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k}) - (3\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k})$ $= -7\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ $\overrightarrow{SR} = \overrightarrow{PQ}$ (opposite sides of a parallelogram are equal) $= -7\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ $\overrightarrow{OS} = \overrightarrow{OR} - \overrightarrow{SR}$ $= (3\underset{\sim}{i} + 4\underset{\sim}{j} - 3\underset{\sim}{k}) - (-7\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k})$ $= 10\underset{\sim}{i} + 5\underset{\sim}{j} - 4\underset{\sim}{k}$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Finds $\overrightarrow{PQ}$ or shows some understanding.
14(a)	$t = \tan x \ \therefore \sin x = \dfrac{t}{\sqrt{1+t^2}}, \cos x = \dfrac{1}{\sqrt{1+t^2}}$ $dt = \sec^2 x\, dx = (1 + t^2)\, dx$ $dx = \dfrac{1}{1+t^2} dt$ When $x = 0$ then $t = 0$ and when $x = \frac{\pi}{4}$ then $t = 1$ $3\sin^2 x + 5\cos^2 x = 5 - 2\sin^2 x$ $\int_0^{\frac{\pi}{4}} \dfrac{1}{3\sin^2 x + 5\cos^2 x} dx = \int_0^1 \dfrac{1}{5 - 2(\frac{t^2}{1+t^2})} \times \dfrac{2}{1+t^2} dt$ $= \int_0^1 \dfrac{1}{5+3t^2} dt$ $= \left[\dfrac{1}{\sqrt{15}} \tan^{-1} \sqrt{\dfrac{5}{3}} t\right]_0^1$ $= 0.2354 \ldots \approx 0.235$ (3 s.f.)	4 Marks: Correct answer. 3 Marks: Correct expression for the integral in terms of t 2 Marks: Finds the value of $3\sin^2 x + 5\cos^2 x$ in terms of t and changes the limits. 1 Mark: Sets up the integration using t formulas.

14(b) (i)	$x = 1 + \sqrt{2}\cos\left(t - \frac{\pi}{4}\right)$ or $\sqrt{2}\cos\left(t - \frac{\pi}{4}\right) = x - 1$ $\dot{x} = -\sqrt{2}\sin\left(t - \frac{\pi}{4}\right)$ $\ddot{x} = -\sqrt{2}\cos\left(t - \frac{\pi}{4}\right)$ $\ddot{x} = -(x - 1)$	1 Mark: Correct answer.
14(b) (ii)	$x = 1 + \sqrt{2}\cos\left(t - \frac{\pi}{4}\right)$ At point O $x = 0$ $\cos\left(t - \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ $t - \frac{\pi}{4} = \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}, \ldots$ $t = \pi, \frac{3\pi}{2}, 3\pi, \ldots$ $\therefore$ First passes through O after π seconds	2 Marks: Correct answer. 1 Mark: Finds the value of $\cos\left(t - \frac{\pi}{4}\right)$.
14(b) (iii)	$x = 1 + \sqrt{2}\cos\left(t - \frac{\pi}{4}\right)$ (in the form $x = a\cos(nt + \alpha) + c$) Amplitude is $\sqrt{2}$ and period is 2π Average speed is the time taken to complete one oscillation. Average speed $= \frac{\text{Distance travelled}}{\text{Time taken}}$ $= \frac{4a}{T}$ $= \frac{4 \times \sqrt{2}}{2\pi}$ $= \frac{2\sqrt{2}}{\pi}$ ms^{-1}	2 Marks: Correct answer. 1 Mark: Makes some progress.
14(c) (i)	$w = 2 + bi$ $\arg w = \tan^{-1}\frac{b}{2} = \frac{1}{4}\pi$ $\frac{b}{2} = 1$ $\therefore b = 2$	1 Mark: Correct answer.
14(c) (ii)	$\lvert z \rvert = \sqrt{\left(\sqrt{2}\right)^2 + (4)^2} = \sqrt{18} = 3\sqrt{2}$ $\lvert w \rvert = \sqrt{2^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$ $\lvert zw \rvert = \lvert 3\sqrt{2} \times 2\sqrt{2} \rvert$ $= 12$	1 Mark: Correct answer.

14(c) (iii)	$16\lvert z^n\rvert = 81\lvert w^n\rvert$ $16\times(\sqrt{18})^n = 81\times(\sqrt{8})^n$ $\frac{(\sqrt{18})^n}{(\sqrt{8})^n} = \frac{81}{16}$ $\left(\frac{9}{4}\right)^{\frac{1}{2}n} = \frac{81}{16}$ $= \left(\frac{9}{4}\right)^2$ $\frac{1}{2}n = 2$ $\therefore n = 4$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
15(a) (i)	$P(4,i)$ and $Q(1,4i)$ represent $z_1 = 4+i$ and $z_2 = 1+4i$. Point R is constructed by completing the parallelogram. $z_1 + z_2 = 5 + 5i$ Not to scale	2 Marks: Correct answer. 1 Mark: Constructs an Argand diagram containing z_1 and z_2.
15(a) (ii)	*OPQR* is a rhombus. Parallelogram with $OP = OQ = \sqrt{17}$	1 Mark: Correct answer.
15(b) (i)	$I_n = \int x^n e^x dx$ $= x^n e^x - \int e^x n x^{n-1} dx$ $= x^n e^x - n\int e^x x^{n-1} dx$ Show that $I_n = x^n e^x - nI_{n-1}$ for $n = 1, 2, 3, \ldots$	2 Marks: Correct answer. 1 Mark: Sets up the integration by parts.
15(b) (ii)	$I_2 = x^2 e^x - 2I_1$ $I_1 = xe^x - I_0$ Now $I_0 = \int x^0 e^x dx = e^x$ $\int_0^2 x^2 e^x dx = \left[x^2 e^x - 2\left(xe^x - (e^x)\right)\right]_0^2$ $= (4e^2 - 2[(2e^2 - 0) - (e^2 - 1)])$ $= 2e^2 - 2$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
15(c) (i)	Horizontal $\ddot{x} = 0$ $\dot{x} = c_1$ $\dot{x} = V\cos\theta$ $x = Vt\cos\theta + c_2$ When $t = 0$ then $x = 0$	2 marks: Correct answer. 1 mark: Finds the horizontal or the vertical displacements.

	$x = Vt\cos\theta$ Vertical $\ddot{y} = -10$ $\dot{y} = -10t + c_3$ When $t = 0$ then $\dot{y} = V\sin\theta$ $\dot{y} = -10t + V\sin\theta$ $y = -5t^2 + Vt\sin\theta + c_4$ When $t = 0$ then $y = 0$ $\dot{y} = -10t + V\sin\theta$ $y = -5t^2 + Vt\sin\theta$	
15(c) (ii)	$x = Vt\cos\theta$ ① $y = -5t^2 + Vt\sin\theta$ ② Making t the subject of equation ① $t = \dfrac{x}{V\cos\theta}$ Subject this result into equation ② $y = -5 \times \dfrac{x^2}{V^2\cos^2\theta} + V \times \dfrac{x}{V\cos\theta} \times \sin\theta$ $y = x\tan\theta - \dfrac{5x^2}{V^2}\sec^2\theta$	2 Marks: Correct answer. 1 Mark: Finds t in terms of x and substitutes into the equation for y.
15(c) (iii)	 Using the result in part(ii) $y = x\tan\theta - \dfrac{5x^2}{V^2}\sec^2\theta$ $2 = 25\tan\theta - \dfrac{5 \times 25^2}{60^2}(1 + \tan^2\theta)$ $288 = 3600\tan\theta - 125(1 + \tan^2\theta)$ $125\tan^2\theta - 3600\tan\theta + 413 = 0$ $\tan\theta = \dfrac{3600 \pm \sqrt{3600^2 - 4 \times 125 \times 413}}{2 \times 125}$ $\theta = 2°$ or $88°$ $\therefore$ Angle of projection is $2°$ or $88°$	2 Marks: Correct answer. 1 Mark: Substitutes correct values for x, y and V into the cartesian equation of motion.
15(d)	Statement: if $(x-1)^2$ is odd then x is even. Contrapositive: if x is not even then $(x-1)^2$ is not odd ie: if x is odd then $(x-1)^2$ is even If x is odd $\exists\, k, k \in \mathbb{Z}$ *where* $x = 2k + 1$ $\therefore (x-1)^2 = ((2k+1)-1)^2 = (2k)^2 = 4k^2$ which is even $\therefore$ by contraposition, statement is true	2 Marks: Correct answer. 1 Mark: shows some understanding.

16(a) (i)	$\lvert z\rvert = \dfrac{1}{2}$	1 Mark: Correct answer.
16(a) (ii)	$\dfrac{1}{1-z} = \dfrac{1}{1-\frac{1}{2}(\cos\theta + i\sin\theta)}$ $= \dfrac{2}{2-\cos\theta - i\sin\theta}$ $= \dfrac{2(1-\cos\theta + i\sin\theta)}{(2-\cos\theta)^2 - (i\sin\theta)^2}$ $= \dfrac{2(1-\cos\theta) + 2i\sin\theta}{5-4\cos\theta}$ $\therefore Im\left(\dfrac{1}{1-z}\right) = \dfrac{2\sin\theta}{5-4\cos\theta}$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Shows some understanding of De Moivre's theorem.
16(b) (i)	$F = ma$ $50g - 0.05v = 50 \times a$ $a = g - 0.001v$  $a = g - 0.001v$ $\frac{dv}{dt} = g - 0.001v \therefore \frac{dt}{dv} = \frac{1}{g-0.001v}$ $t = \displaystyle\int \frac{1}{g-0.001v}\,dv$ $t = -1000\ln(g-0.001v) + C$ Now $v = 0$ when $t = 0$ then $C = 1000\ln g$ $\therefore t = 1000\ln\left(\dfrac{g}{g-0.001v}\right)$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Finds $a = g - 0.001v$ or draws a diagram to resolve forces.
16(b) (ii)	$t = 1000\ln\left(\dfrac{g}{g-0.001v}\right)$ $e^{0\cdot001t} = \dfrac{g}{g-0.001v}$ $g - 0.001v = ge^{-0\cdot001t}$ $\therefore v = 1000g(1-e^{-0\cdot001t})$	1 Mark: Correct answer.
16(b) (iii)	Terminal velocity occurs when the acceleration is equal to zero. $a = g - 0.001v$ $0 = g - 0.001v$ $v = 1000g$ $\therefore$Terminal velocity is $1000g$.	1 Mark: Correct answer.
16(b) (iv)	$v = \dfrac{dx}{dt} = 1000g(1-e^{-0\cdot001t})$ $x = \displaystyle\int (1000g - 1000ge^{-0\cdot001t})\,dt$ $= 1\,000gt + 1\,000\,000ge^{-0\cdot001t} + C$ When $t = 0$ then $x = 0$	2 Marks: Correct answer. 1 Mark: Finds the correct expression for x.

	$C = -1\,000\,000g$ $x = 1\,000gt + 1\,00\,000ge^{-0\cdot001t} - 1\,000\,000g$ $\therefore x = 1\,000gt + 1\,000\,000g(e^{-0\cdot001t} - 1)$	
16(c)	If $x = \frac{p}{q}$ is a root of $ax^3 - 3x + b = 0$ then $a(\frac{p}{q})^3 - 3(\frac{p}{q}) + b = 0$ $\therefore ap^3 - 3pq^2 + bq^3 = 0$ $\therefore ap^3 = 3pq^2 + bq^3$ ① or $bq^3 = 3pq^2 - ap^3$ ② In ①, $3pq^2 + bq^3$ is an integer and q has no factors in common with p, so q divides a. In ②, $3pq^2 - ap^3$ is an integer and p has no factors in common with q, so p divides b. If $x = \frac{p}{q}$ is a rational root of $x^3 - 3x - 1 = 0$, then p divides -1 and q divides 1. Then the only possibilities are $x = \pm 1$. Substituting $x = \pm 1$ into $x^3 - 3x - 1 = 0$: $x = 1$: $1 - 3 - 1 \neq 0$ $x = -1$: $-1 + 3 - 1 \neq 0$ Since neither work $x^3 - 3x - 1 = 0$ does not have rational roots.	4 Marks: Correct answer. 3 Marks: Does not show both cases of $x = \pm 1$ 2 Marks: Shows some attempt of a rational number proof 1 Mark: Correct substitution of $x = \frac{p}{q}$